International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 3 Issue 6 ǁ June 2015 ǁ PP.28-34

Kinematic and Dynamic Analysis of 6-UPS Parallel Robot

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ABSTRACT: *Kinematics and dynamics analysis of a 6-DOF parallel robot is performed. The mechanism of this parallel robot is composed of a moving platform and a static platform, the platform are linked by 6-UPS branched chains. The position and pose of the robot moving platform are totally described by 6 variables, including 3 displacement and 3 angles of a reference point on the platform. Firstly, the constraint equations between the 6 pose parameters of the robot manipulator are deduced, and the analysis of the variables are given. Then, the dynamic equations of the parallel robot are established based on Lagrange equation. Based on this, the variations of angular velocity, driving force / torque and energy consumption of the component are analyzed by an example. The analysis is very important for the study of dynamic performance analysis, optimal design and control of the parallel mechanism.*

Key words：*Parallel manipulator; Kinematics; Dynamics; Position*

I. INTRODUCTION

The particularity of parallel robot structure makes it have the advantages which the serial robot doesn't have, This has caused wide attention of the international academic community. Most 6-DOF parallel robot is based on the structure of Stewart platform, 6-DOF parallel robot has the following advantages: it can meet the needs of most industrial operations, mechanism of the complexity and the cost is low, kinematics and dynamics model is relatively simple, and the control is easy. Therefore, the 6-DOF parallel robot has broad application prospect. Such as 1983, HUNT [1] proposed the 6-DOF mechanism, it gets a wide range of applications because of its two rotations and a mobile. LEE[2-3] analyzed kinematic and dynamic of 6-DOF mechanism, and they used the mechanism as the main arm of 6-DOF robot manipulator. The optimal design of kinematic for planar 6-DOF parallel manipulator was analyzed by GOSSELIN[4].The differential kinematics of 6-DOF parallel manipulator is studied by FANG[5-6].The instantaneous motion of 6-DOF is analyzed by using the screw theory. The position of the 6-DOF mechanism based on the symmetric structure of the 6-DOF parallel robot was studied by FANG[7-8]. Based on the relationship of system differential movement, Li Jianfeng [8] analyzed the kinematics and dynamics of the parallel mechanism of 6-DOF. The kinematics of the 6-DOF parallel robot was analyzed by CARRETERO[9], and the parameters of the system were optimized by using the nonlinear optimization method. WANG[10]studied the static balance problem of 6-DOF parallel robot by adding weights and springs. However, the types and the number of the parallel mechanism are so much, the kinematics and dynamics of the parallel mechanism is still insufficient.

Based on the kinematic characteristics of a space 6-DOF parallel robot the constraint equations and the pose relationship of the parallel mechanism are analyzed and the equations of the 6 pose variables are given. The dynamic equations of 6-DOF parallel manipulator are derived by Lagrange equation, and then the dynamic characteristic of the robot is analyzed.

II. KINEMATIC ANALYSIS OF 6-DOF PARALLEL ROBOT

As shown in figure 1,it's schematic diagram of a 6-DOF parallel robot. It consists of a moving platform P1P2P3, three branched BiCiPi (i=1, 2, 3) and a static platform (base) B1B2B3.the moving platform is connected by spherical vice (S deputy) and the branch chain. The static platform is connected with each branch by the rotating deputy (R),And the axis of the rotational pairs of Bi is parallel to the axis of Ci (i=1, 2, 3).Respectively establish local (dynamic) coordinate system Pxyz which is consolidated with the moving platform and system (fixed) coordinate system OXYZ,As shown in Figure 1, the origin of the coordinate system P and O are located at the geometrical center of the moving platform and the static platform, The axis Z and Z are perpendicular to the dynamic and the static platform and they are upward, The axis X, y and X, Y are respectively parallel and vertical the edge P2P3 and B2B3 of the upper and lower platforms. The Xi axis of local coordinate system Bixiyizi (i=1, 2, 3) is the same to Bi rotating sub axis, Zi is vertical the static platform B1B2B3 and it's upward, while the axis of Yi is perpendicular to the axis of Xi and Zi.

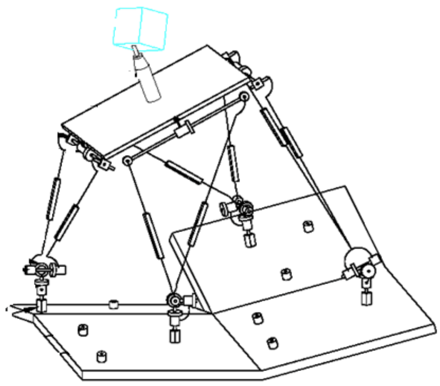

Fig. 1 Schematic diagram of 6-DOF parallel robot

The 6-DOF parallel robot moving platform and the static platform are all Squares, and the distance of the geometrical center of the moving and static platform is *l ppi* =r, *lOBi* =R(i=1, 2, 3).then, under the system coordinate system OXYZ, coordinates of Bi (i=1, 2, 3) on the static platform are

$$B_1 = \begin{pmatrix} 0 \\ R \\ 0 \end{pmatrix} \quad B_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2}R \\ -\frac{1}{2}R \\ 0 \end{pmatrix} \quad B_2 = \begin{pmatrix} \frac{\sqrt{3}}{2}R \\ -\frac{1}{2}R \\ 0 \end{pmatrix} \tag{1}$$

Similarly, in the local (dynamic) coordinate system Pxyz, coordinates of spherical Pi (i=1, 2, 3) in dynamic platform are

$$p_1 = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \quad p_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2}r \\ -\frac{1}{2}r \\ 0 \end{pmatrix} \quad p_2 = \begin{pmatrix} \frac{\sqrt{3}}{2}r \\ -\frac{1}{2}r \\ 0 \end{pmatrix} \tag{2}$$

From the local dynamic coordinate system Pxyz to the system (stationary) coordinate system OXYZ. Set transformation matrix from the local dynamic coordinate system Pxyz to the system (stationary) coordinate system OXYZ is

$$T=\begin{pmatrix} n_i & o_i & a_i & X_p \\ n_j & o_j & a_j & Y_p \\ n_k & o_k & a_k & Z_p \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

Under the system coordinate system OXYZ, the coordinates of Pi (i=1, 2, 3) points in spherical hinge center of the moving platform can be expressed as

$$\begin{pmatrix} P_i \\ 1 \end{pmatrix}_{XYZ} = T\begin{pmatrix} p_i \\ 1 \end{pmatrix}_{xyz} \tag{4}$$

As the three branches B2C2P2, B3C3P3 and B1C1P1 of the parallel mechanism are respectively restricted by the two rotating pairs. So the moving tracks of P1, P2 and P3 points in the center of the ball hinge of this parallel robot moving platform P1P2P3 can only be located in three vertical plane, Based on these, the three constraint equations of the system are

$$\begin{cases} X=0 \\ Y=\dfrac{\sqrt{3}}{3}X \\ Y=-\dfrac{\sqrt{3}}{3}X \end{cases} \tag{5}$$

Combined (2) ~ (5) and then simplification, the constraint equations of the mechanism are

$$X_P=-o_i r \qquad Y_p=\frac{r}{2}\left(o_j-n_i\right) \qquad o_i=n_i \tag{6}$$

Z-Y-X Euler angle (α, β, γ) is expressed in the attitude of the mobile platform[11] P1P2P3,the type (3) can be expressed as

$$T=\begin{pmatrix} \cos\alpha\cos\beta & T_1 & T_3 & X_p \\ \sin\alpha\sin\beta & T_2 & T_4 & Y_p \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma & Z_p \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

Combined (6) ~ (7)

$$\begin{cases} X_P=-r\sin\alpha\cos\beta \\ Y_p=\dfrac{r}{2}\left(\sin\alpha\sin\beta\sin\gamma+\cos\alpha\cos\gamma-\cos\alpha\cos\beta\right) \end{cases} \tag{8}$$

$$\alpha=\arctan\left(\sin\beta\sin\gamma/\left(\cos\beta+\cos\gamma\right)\right) \tag{9}$$

III. DYNAMIC ANALYSIS OF 6-DOF PARALLEL ROBOT

Set the angle between the axis OX and OBi in the 6-DOF parallel mechanism is$\theta 0i$ (i=1, 2, 3). Moving platform point *Pi* (i=1, 3, 2) on the coordinates in local coordinate system Pxyz are $\boldsymbol{pi}=(xpi\ ypi\ 0)^T$.Then, according to the different representations of the points *Pi*(i=1, 2, 3) in the coordinate system OXYZ, we can get the formula (10) (i=1, 2, 3).

$$\begin{cases} \left(R+l_{i1}\cos\theta_{i1}+l_{i2}\cos\theta_{i2}\right)\cos\theta_{0i}=x_{Pi}\cos\alpha\cos\beta+y_{Pi}\left(\cos\alpha\sin\beta\sin\gamma-\sin\alpha\cos\gamma\right)+X_P \\ \left(R+l_{i1}\cos\theta_{i1}+l_{i2}\cos\theta_{i2}\right)\sin\theta_{0i}=x_{Pi}\sin\alpha\cos\beta+y_{Pi}\left(\sin\alpha\sin\beta\sin\gamma+\cos\alpha\cos\gamma\right)+Y_P \\ l_{i1}\sin\theta_{i1}+l_{i2}\sin\theta_{i2}=-x_{Pi}\sin\beta+y_{Pi}\cos\beta\sin\gamma+Z_P \end{cases} \tag{10}$$

Under the system coordinate system OXYZ, the component BiCi is homogeneous in BiCiPi (i=1, 2, 3) and the centroid coordinates is($xi1c$ $yi1c$ $zi1c$) ^{T}, the component CiPi is homogeneous and the centroid coordinates is ($xi2c$ $yi2c$ $zi2c$) ^{T}, then

$$\begin{cases} x_{11c} = 0 \\ y_{11c} = R + \frac{1}{2} l_{11} \cos\theta_{11} \\ z_{11c} = \frac{1}{2} l_{11} \sin\theta_{11} \end{cases} \tag{11}$$

$$\begin{cases} x_{12c} = 0 \\ y_{12c} = R + l_{11} \cos\theta_{11} + \frac{1}{2} l_{12} \cos\theta_{12} \\ z_{12c} = l_{11} \sin\theta_{11} + \frac{1}{2} l_{12} \sin\theta_{12} \end{cases} \tag{12}$$

$$\begin{cases} x_{21c} = \left(R + \frac{1}{2} l_{21} \cos\theta_{21} \right) \cos\theta_{02} \\ y_{21c} = \left(R + \frac{1}{2} l_{21} \cos\theta_{21} \right) \sin\theta_{02} \\ z_{21c} = \frac{1}{2} l_{21} \sin\theta_{21} \end{cases} \tag{13}$$

$$\begin{cases} x_{21c} = \left(R + \frac{1}{2} l_{21} \cos\theta_{21} \right) \cos\theta_{02} \\ y_{21c} = \left(R + \frac{1}{2} l_{21} \cos\theta_{21} \right) \sin\theta_{02} \\ z_{21c} = \frac{1}{2} l_{21} \sin\theta_{21} \end{cases} \tag{14}$$

$$\begin{cases} x_{31c} = \left(R + \frac{1}{2} l_{31} \cos\theta_{31} \right) \cos\theta_{03} \\ y_{31c} = \left(R + \frac{1}{2} l_{31} \cos\theta_{31} \right) \sin\theta_{03} \\ z_{31c} = \frac{1}{2} l_{31} \sin\theta_{31} \end{cases} \tag{15}$$

$$\begin{cases} x_{32c} = \left(R + l_{31} \cos\theta_{31} + \frac{1}{2} l_{32} \cos\theta_{32} \right) \cos\theta_{03} \\ y_{32c} = \left(R + l_{31} \cos\theta_{31} + \frac{1}{2} l_{32} \cos\theta_{32} \right) \sin\theta_{03} \\ z_{31c} = l_{31} \sin\theta_{31} + \frac{1}{2} l_{32} \sin\theta_{32} \end{cases} \tag{16}$$

The centroid speed of the component BiCi and CiPi (i=1,2,3) are respectively $v_{i1} = \left(\dot{x}_{ijc} \quad \dot{y}_{ijc} \quad \dot{z}_{ijc} \right)^T$ and $v_{i1} = \left(\dot{x}_{ijc} \quad \dot{y}_{ijc} \quad \dot{z}_{ijc} \right)^T$,the moment of inertia of the center of mass are recorded as Ji1 and Ji1;the inertia matrix of the moving platform P1P2P3 relative to the Pxyz system is Ip (the inertia of the main moment of inertia is Jx, Jy, Jz) ,the velocity and angular velocity of the moving platform are respectively $v_p = \left(\dot{X}_p \quad \dot{Y}_p \quad \dot{Z}_p \right)^T$ and $w_p = \left(\dot{\gamma} \quad \dot{\beta} \quad \dot{\alpha} \right)^T$ .The zero potential energy surface position of gravity is taken from the plane OXY, the acceleration of gravity was g`, and the elasticity and friction of the member were not neglected. Then the kinetic energy T and the potential energy V can be expressed as

$$T=\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{2}\left[m_{ij}\left(\dot{x}^2{}_{ijc}+\dot{y}^2{}_{ijc}+\dot{z}^2{}_{ijc}\right)+J_{ij}\dot{\theta}^2{}_{ij}\right]+\frac{1}{2}\left(m_0 v^T{}_p v_p+w^T{}_p I_p w_p\right) \tag{17}$$

$$V=\sum_{i=1}^{3}\sum_{j=1}^{2}m_{ij}gz_{ijc}+m_0 gZ_p \tag{18}$$

Type (11) ~ (16) of t derivative, then into type (20),we can get

$$T=\frac{1}{2}\left(\hat{J}_{11}\dot{\theta}^2{}_{11}+\hat{J}_{22}\dot{\theta}^2{}_{21}+\hat{J}_{33}\dot{\theta}^2{}_{31}\right)+\hat{J}_{12}\dot{\theta}_{11}\dot{\theta}_{21}+\hat{J}_{13}\dot{\theta}_{11}\dot{\theta}_{31}+\hat{J}_{23}\dot{\theta}_{21}\dot{\theta}_{31} \tag{19}$$

Substituting(18), (19) to Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}_{i1}}\right)-\frac{\partial T}{\partial\theta_{i1}}+\frac{\partial T}{\partial\theta_{i1}}=\tau_i \qquad i=1,2,3$$

By the analysis, the energy consumption of 6-DOF parallel robot system can be expressed as

$$E=\int_{t_0}^{t_f}\left(\sum_{i=1}^{3}\left|\tau_i\dot{\theta}_{i1}\right|\right)dt \tag{20}$$

IV. CASE STUDY OF 6-DOF PARALLEL ROBOT

System parameters: homogeneous mechanism components are steel, the densityρ=7800kg/m^3;Member length $li1=li2=0.16$m (i=1, 2, 3), rectangular cross section, thickness h=0.010 m, width b=0.010 m;The quality of the mobile platform is m0=0.25kg ,Jx=0.0174 kg·m^2, Jy=0.000 45 kg·m^2, Jz= 0.017 9 kg·m^2, r=0.10 m, R=0.12 m, t0=0 s, tf=10 s.

the law of the system is

$$\begin{cases} x_{32c}=\left(R+l_{31}\cos\theta_{31}+\frac{1}{2}l_{32}\cos\theta_{32}\right)\cos\theta_{03} \\ y_{32c}=\left(R+l_{31}\cos\theta_{31}+\frac{1}{2}l_{32}\cos\theta_{32}\right)\sin\theta_{03} \\ z_{31c}=l_{31}\sin\theta_{31}+\frac{1}{2}l_{32}\sin\theta_{32} \end{cases} \tag{21}$$

In the formula, $s(t)=\frac{t}{t_f}-\frac{1}{2\pi}\sin\frac{2\pi t}{t_f} \qquad t_0\le t\le t_f$ (22)

For the system given motion law, Inverse solution through system kinematics, It can be obtained from the position of each member in this parallel mechanism. In Figure 2, the variation curves of the angular velocity of the driving member BiCi (i=1,2,3) in the 6-DOF parallel robot are given.

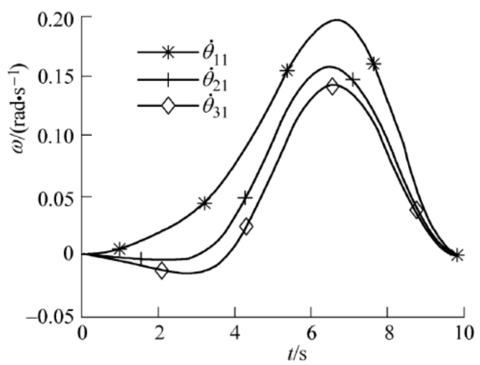

Fig. 2 Driving component angular velocity curve chart

For the system given motion law, The pose and speed of motion of the 6-DOF parallel robot are changed regularly, as shown in Figure 3 and 4.

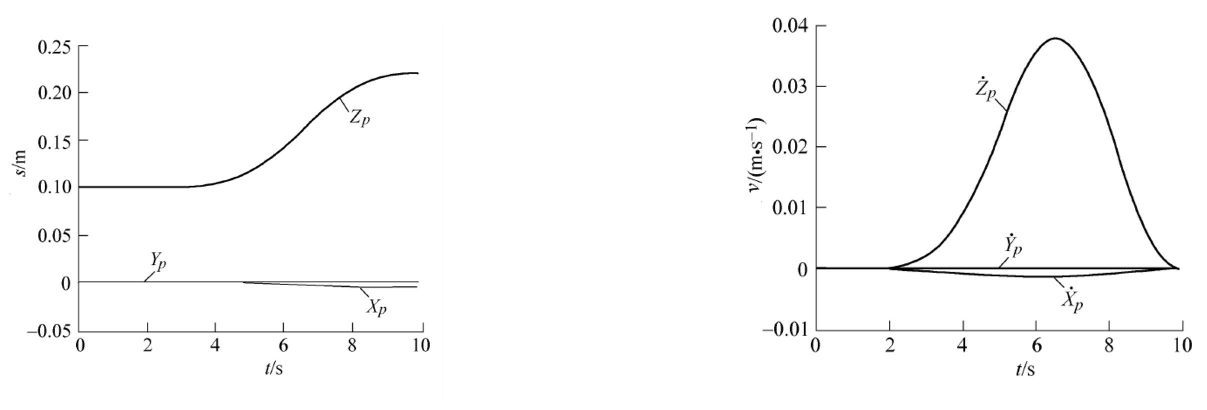

Fig. 3 Moving platform linear displacement and velocity curve chart

Fig. 4 Dynamic platform attitude change curve chart

For the given motion law,the change curve of the driving moment of each active component in the system, as shown in Figure 5.

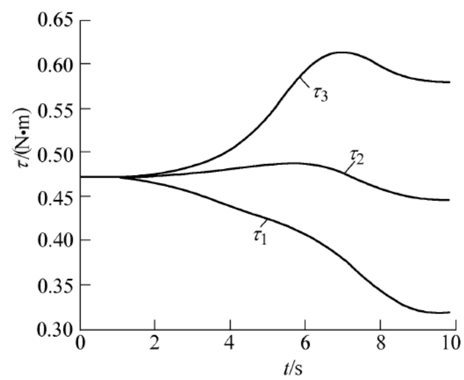

Fig. 5 Drive torque curve chart

V. THE ANALYSIS OF THE RESULT

(1) Zp is the only completely independent variable, It is unconnected with the other five parameters Xp、Yp、α、β、γ.

(2) the Mechanism has 3 parameters that could be arbitrarily selected, but it must include the Zp, the other two parameters can be arbitrarily selected.

(3) when β=0 or γ=0 or β=0 and γ=0, then α =0; whenβ≠0 and γ≠0, then α≠0;At least one of the β and γ is zero when α=0.

(4) when α=0 or β=90º, Xp=0.

In this paper, the dynamic model of 6-DOF parallel robot is established by using Lagrange equation. The pose of the system and the movement curves of the drivers are given. The change law of the driving force / torque and energy consumption are analyzed. These contents are very important for the study of the dynamic performance of 6-DOF robot, the optimization design of mechanism and the realization of the control law of the system.

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